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During the past three years there have been 14 journal publications and 6 chapters published in books in addition to 4 Program in Applied Mathematics Reports (preprints of the Program) which have been submitted for publication. Also, the PI has been invited to give 30 lectures at universities and international conferences throughout the world. This has been a very active and productive period in this research program. A brief overview and description of our research activity is given below. Full details are included in the publications.

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Final Report

Abstract

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Project Description

Research investigations involving the fundamental understanding and applications of nonlinear wave motion and related studies of inverse scattering and numerical computation have been carried out and a number of significant results have been obtained. The main results are summarized below.

A class of nonlinear wave equations which can be solved by the Inverse Scattering Transform (IST) have been studied. Equations which we have investigated include the following physically important equations.

i) The Kadaomtsev-Petviashvili (KP) equation, which is a 2+1 dimensional analog of the well known Korteweg-deVries (KdV) equation and arises in the study of nonlinear waves in shallow water, long internal waves, plasma waves etc.;

ii) the Davey-Stewartson equation which is a 2+1 analog of the nonlinear Schrödinger equation and arises in the study of nonlinear modulations of deep water waves, and nonlinear waves in optical fibers;

iii) and the 2+1 Toda system, which is fundamental in the study of lattice dynamics.

The solutions obtained by IST correspond to the Cauchy initial value problem with decaying initial data. General solutions are obtained to the initial value problem as well as a special, though very important class of solutions referred to as lump type soliton solutions. These soliton solutions decay in all spatial directions. We have also investigated the solution of certain types of boundary value problems. The direct and inverse scattering analysis used in the IST method leads to a system of linear integral equations which follow from a coupled DBAR-nonlocal Riemann-Hilbert problem. In the initial value problem the time dependence of the scattering data is elementary. In the case of the boundary value problem, we have analyzed various cases, one of which requires the introduction of a suitable radiation condition in order to obtain a unique solution. This radiation condition is a nonlinear generalization of the well-known Sommerfeld radiation condition frequently employed in linear problems (such as the Helmholz equation).

We have also solved two important systems via the IST method: a "Volterra" system in 2+1 dimensions, which arises in plasma physics and a new one dimensional nonlinear equation which we refer to as the Toda differential-delay equation. The scattering analysis in the latter case is novel in that the inverse problem involves developing and analyzing a Riemann-Hilbert problem posed on an infinite number of strips in the complex plane.

An important question to resolve when dealing with nonlinear wave problems, is whether there might be wave collapse; i.e. will the solution blow up in finite time. For example, in the case of the cubic nonlinear Schrödinger equation, it is known that collapse occurs in dimensions greater than two. We have shown that generalized nonlinear KP equations have unstable solitary wave solutions and are likely to collapse when the power of the polynomial nonlinearity is greater than or equal to two. We have also shown that the solution grows rapidly and blow up occurs in concrete numerical simulations.

Research in the study of computational chaos in moderate to long time numerical simulations continues. We are considering the well-known integrable one dimensional nonlinear soliton equations including the nonlinear Schrödinger, sine-Gordon and modified KdV equations with periodic initial values. In certain parameter regimes we have seen computational chaos generated by truncation errors as well as tiny errors even on the order of roundoff. We have explained this phenomena in terms of proximity to existing homoclinic manifolds of the nonlinear wave equations. We have found that the initial data can be easily chosen so that either effect: truncation or roundoff, will be capable of creating a chaotic response, even though the governing equation itself is not chaotic. The roundoff effects are explained by the fact that the initial data is exponentially close to a homoclinic manifold. Although in physical space one cannot "see" the exponential proximity, nevertheless when one employs the IST method, it is transparent, after a (nontrivial) calculation. The IST method also provides natural and effective diagnostics by which different computational methods can be compared.

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PAM†: Program in Applied Mathematics Report